OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4763/01

MATHEMATICS (MEI)

Mechanics 3

QUESTION PAPER

WEDNESDAY 23 MAY 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4763/01 sent with the standard paper
MEI Examination Formulae and Tables (MF2) sent with the standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SC/TP) 168156/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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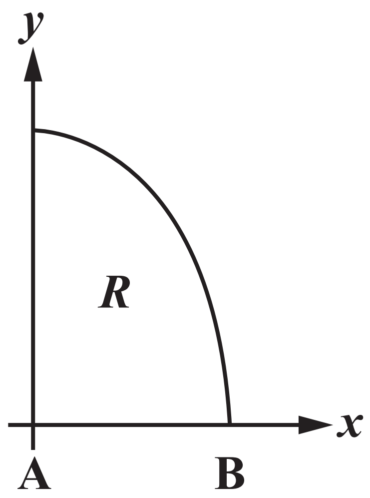
1 **The shaded region R in the xy plane is bounded by the axes and the part of the curve $y = 8 - x^3$ that lies in the first quadrant as shown in Fig. 1. The points A and B on the boundary of R are at the origin and the point where the curve meets the positive x-axis, respectively.**

FIG. 1



A uniform solid is formed by rotating R through one complete revolution about the x-axis.

(i) Find the coordinates of the centre of mass of the solid. [7]

A uniform lamina is made in the shape of R.

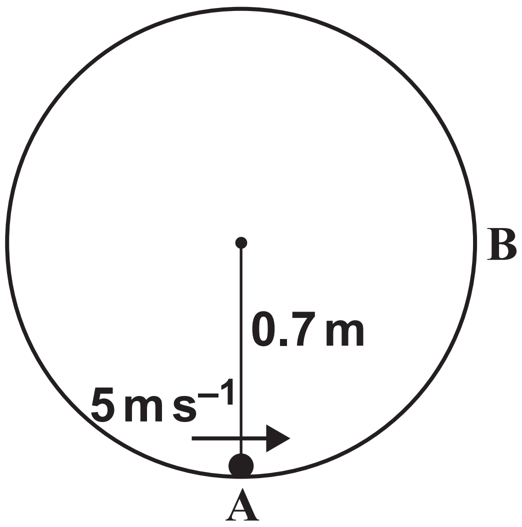
(ii) Show that the coordinates of the centre of mass of the lamina are $\left(\frac{4}{5}, \frac{24}{7}\right)$. [6]

The lamina is suspended freely from the point B.

(iii) Calculate the angle that AB makes with the vertical. [3]

2 A smooth cylindrical pipe of internal radius 0.7 m is fixed in a position with its axis horizontal. A small ball of mass 0.1 kg is inside the pipe and is projected horizontally from the lowest point, A, of the pipe. The ball moves in a vertical plane perpendicular to the axis of the cylinder. The initial speed of the ball is 5 m s^{-1}. The point B is where the ball first reaches the same vertical level as the axis of the pipe. The ball is still in contact with the pipe at B. The cross-section of the pipe in which the ball moves and the positions of A and B are shown in Fig. 2.

FIG. 2



(i) Calculate the speed of the ball when it is at B. Calculate also the normal reaction of the pipe on the ball at B. [5]

The ball leaves the inner surface of the pipe at the point C. It subsequently passes through a point D which is vertically above A.

(ii) Calculate the horizontal and vertical components of the velocity of the ball at C. [10]

(iii) Hence determine the distance AD. [5]

3 A light elastic string AB has natural length 0.8 m and modulus of elasticity 70 N. The end A is attached to a fixed point and the end B is attached to a particle of mass 1.2 kg.

The string and particle hang in equilibrium with B vertically below A.

(i) Show that the stretched length of the string is 0.9344 m. [4]

The particle is now held at a point 1.3 m vertically below A and released from rest. In the subsequent motion the speed of the particle is $v\,\text{m s}^{-1}$ when it is at a height of h m above the release point.

(ii) Show that, during the motion before the string becomes slack, $v^2 = \frac{1}{3}(159.95h - 218.75h^2)$. [6]

(iii) Find an expression for v^2 in terms of h during the motion while the string is slack. [3]

(iv) Calculate the maximum speed of the particle during its motion. [4]

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4 (a) A simple pendulum consists of a light rigid rod AB of length 1.25 m with a mass 0.8 kg attached to the end B and the rod hinged at the end A so that the rod can rotate freely in a vertical plane. The rod is held at rest with AB making an angle 0.1 radians with the downward vertical, and released from rest.

(i) Show that the motion of the pendulum approximates to simple harmonic motion with period $\frac{5}{7}\pi$ seconds. [6]

(ii) Calculate the angular speed of the pendulum when it has turned through 0.05 radians from its initial position. [2]

(iii) Calculate the time the pendulum takes to turn through 0.05 radians from its initial position. [2]

(b) (i) Show that the dimensions of moment of force and the dimensions of kinetic energy are the same. [2]

(ii) Given that angles are dimensionless, state the dimensions of angular speed and angular acceleration. [2]

A compound pendulum is formed when a rigid body is free to rotate about a fixed horizontal axis. The equation of motion of the compound pendulum is

moment of weight $= -I\ddot{\theta}$,

where I is the moment of inertia of the compound pendulum and $\ddot{\theta}$ is its angular acceleration.

(iii) Use the equation of motion to deduce that I has dimensions ML^2. [2]

The kinetic energy, T, of the compound pendulum is believed to be given by the formula

$$T = km^{\alpha} I^{\beta} \dot{\theta}^{\gamma},$$

where k is a dimensionless constant, m is the mass of the compound pendulum and $\dot{\theta}$ is its angular speed.

(iv) Use dimensional analysis to determine α, β and γ. [3]

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